

Catalan numbers and additively idempotent semirings

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The purpose of the present talk is to provide a new applications of remarkable Catalan numbers. In Richard Stanley's book *Enumerative Combinatorics*, Volume II (Cambridge University Press) there are many combinatorial objects that are counted by the Catalan numbers as well as applications in graph theory, Young diagrams, lattice theory, real matrices, real polynomials and so on. We show applications of Catalan numbers in some additively idempotent semirings which appeared in my results in 5 papers published between 2011 and 2023.

The set of nilpotent endomorphisms in the endomorphism semiring of a finite chain is a semiring of order $(n - 1)$ th Catalan number and is an ideal in another semiring of order n th Catalan number. The semiring of k th nilpotent endomorphisms, where $0 \leq k \leq n - 1$ is of order a product of two Catalan numbers. By complex products of Catalan numbers we describe the roots of arbitrary idempotent of the endomorphism semiring of a finite chain.

In an additively idempotent semiring which is a generalization of the endomorphism semiring of a finite chain considered as a simplex we prove that the subsimplex of nilpotent elements is of order $(n - 1)$ th Catalan number and it is closed under derivations which are projections of the simplex to some simplices.

In the last paper, 2023, we prove that an additively idempotent semiring which is an S_0 -semialgebra, where S_0 is a commutative additively idempotent semiring, and with a finite basis of a special type is isomorphic to a matrix semiring. As a consequence we obtain two different semirings of upper triangular matrices over the Boolean semiring, which are of order $(n + 1)$ th Catalan number.